BRAIN MAP

ELECTROSTATICS

In dry weather, a spark is produced by walking across certain types of carpet and then bringing one of the fingers near a metal doorknob, or even a person. Multiple sparks can be produced when a sweater or cloth is pulled from the body. It reveals that electric charge is present in our bodies, sweaters, carpets, doorknobs, computers etc. In fact, every object contains a vast amount of electric charge.

Coulomb's Law

- Electric force between two point charges:
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = -\vec{F}_{21}$$
where $K = 1$ for free space, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N m}^2\ \text{C}^{-2}$
- Coulomb's law agrees with Newton's third law.
- Force on charge q_1 due to remaining charges q_2, q_3, q_4, ... q_n in the region.
$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$
It is a vector sum of all the forces acting on point charge q_1.

Electric Field

- Due to a point charge, $\vec{E} = \frac{\vec{F}}{q_{\text{test}}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$
- Due to positive charge, electric field is away from the charge and due to negative charge, it is towards the charge.
- Due to charge distribution,

Linear charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\lambda\Delta L}{r^2}\hat{r}$

Surface charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\sigma\Delta S}{r^2}\hat{r}$

Volume charge, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{\rho\Delta V}{r^2}\hat{r}$

Electric Flux and Gauss's Law

- Electric flux through a plane surface area S held in a uniform electric field
$$\Delta\phi_E = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\theta$$
- According to Gauss's law, electric flux through a closed surface S is equal to $(1/\ _0)$ times charge enclosed.
$$\phi_E = \oint\vec{E}\cdot d\vec{S} = \frac{q_{en}}{\varepsilon_0}$$

Applications of Gauss's Law

- Electric field due to a long straight wire of uniform linear charge density $E = \frac{\lambda}{2\pi\varepsilon_0 r}$
- Electric field due to an infinite plane sheet of uniform surface charge density , $E = \frac{\sigma}{2\varepsilon_0}$
- Electric field due to two positively charged parallel plates with charge densities $_1$ and $_2$ such that $_1 > \ _2 > 0$,

$E = \frac{1}{2\varepsilon_0}(\sigma_1+\sigma_2)$ (Outside the plates)

$E = \frac{1}{2\varepsilon_0}(\sigma_1-\sigma_2)$ (Inside the plates)

- Electric field due to two equally and oppositely charged parallel plates,
$E = 0$ (For outside points), $E = \frac{\sigma}{\varepsilon_0}$ (For inside points)
- Electric field due to a thin spherical shell of charge density and radius R,

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ For $r > R$ (Outside points)

$E = 0$ For $r < R$ (Inside points)

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ For $r = R$ (At the surface)

where, $q = 4\ \ R^2$.

- Electric field of a solid sphere of uniform charge density and radius R

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ For $r > R$ (Outside points)

$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$ For $r < R$ (Inside points)

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ For $r = R$ (At the surface)

where $q = \frac{4}{3}\pi R^3\rho$

Electric Charge

- A physical quantity responsible for electromagnetism. There are only two kinds of charge, positive and negative.
- **Conservation of charge :** The total charge of an isolated system is always conserved.
- **Quantisation of charge :** $Q = ne$
where, $n = \pm1, \pm2, \pm3, \ldots$ and $e = 1.6\times10^{-19}$ C

Electric Potential

- Work done by an external force (equal and opposite to the electric force) in bringing a unit positive charge from infinity to a point = electrostatic potential (V) at that point.
- Due to a point charge: $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
- Potential difference between two points,
$$V_P - V_R = \frac{U_P - U_R}{q} = \frac{W_{RP}}{q}$$
- Potential at a point P, due to system of charges $q_1, q_2, q_3, \ldots$
$$V_P = V_1 + V_2 + V_3 + \ldots = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+\ldots\right)$$
It is algebraic sum of the potentials due to the individual charges.

Relation between Field and Potential

- $E = -\frac{dV}{dr}$
- Electric field is in the direction in which the potential decreases steepest.
- Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

Electrostatics of Conductors

- At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.
- The interior of a conductor can have no excess charge in the static situation.
- Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
- Electric field at the surface of a charged conductor $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Potential of Charge Distributions

Charge distribution	Formula
Uniformly charged spherical shell	$V_i = V_s = \frac{Kq}{R} = \frac{\sigma R}{\varepsilon_0}$, $V_o = \frac{Kq}{r}$
Uniformly charged solid sphere	$V_i = \frac{Kq}{R^3}(1.5R^2 - 0.5r^2)$ $V_s = \frac{Kq}{R}$, $V_o = \frac{Kq}{r}$
On the axis of uniformly charged ring	$V = \frac{Kq}{\sqrt{R^2+x^2}}$ At centre, $x = 0\ \therefore\ V = \frac{Kq}{R}$
Infinitely long line charge	$PD = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)$

Electric Potential Energy

- Electric potential energy of a system of two point charges, $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$
- Electric potential energy of a system of N point charges, $U = \frac{1}{4\pi\varepsilon_0}\sum_{\substack{\text{all pairs}\\(j>k)}}\frac{q_j q_k}{r_{jk}}$

Electric Dipole

- A pair of equal and opposite charges q and $-q$ separated by a small distance $2a$.
- The direction of dipole is the direction from $-q$ to q.
- Dipole moment, $p = q\times 2a$
- Electric Potential, $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$
- Dipole field at an axial point at distance r from the centre of the dipole $E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$
when $r >> a$ $E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$
- Dipole field at an equatorial point $E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$
at distance r from the centre of the dipole is
when $r >> a$ $E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$
- Torque, $\tau = pE\sin\theta$
- Potential energy of an electric dipole in a uniform electric field, $U = -pE(\cos\ _2 - \cos\ _1)$
- ➢ If initially the dipole is perpendicular to the field E, $_1 = 90°$ and $_2 =$, then $U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$
- ➢ If initially the dipole is parallel to the field E, $_1 = 0°$ and $_2 =$, then $U = -pE(\cos\ -1) = pE(1-\cos\ )$

Dielectric and Polarisation

- A dielectric develops a net dipole moment in an external electric field.
- Polarisation = $\frac{\text{net dipole moment}}{\text{volume}}$
For a linear isotropic dielectric, $\vec{P} = \chi_e\vec{E}$
- Surface charge density due to polarisation,
$$\sigma_P = \vec{P}\cdot\hat{n} = P\cos\theta$$

Capacitors and Capacitance

- A capacitor is a system of two conductors separated by an insulator.
- Capacitance, $C = \frac{q}{V_o} = \frac{q}{V_1 - V_2}$
- For a spherical conductor, $C = 4\ \ _0 R$
- Energy stored in a charged conductor or capacitor,
$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Different Types of Capacitors

- Capacity of parallel plate capacitor, $C = \frac{K\varepsilon_0 A}{d}$
- Induced charges, $\sigma_i = \sigma\left(1-\frac{1}{K}\right)$ or $q_i = q\left(1-\frac{1}{K}\right)$
- Capacity of capacitor partially filled with a dielectric, $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$
- Force of attraction between plates of capacitors, $F = \frac{q^2}{2A\varepsilon_0}$
- Capacity of spherical capacitor, $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$
- Capacity of cylindrical capacitor per unit length, $\frac{2\pi\varepsilon_0}{\ln(b/a)}$

Combination of Capacitors

- Capacitors in series: $\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots = \sum_i\frac{1}{C_i}$
- Capacitors in parallel: $C = C_1 + C_2 + C_3 + \ldots = \ _i C_i$

Van de Graaff Generator

- By means of a moving belt and suitable brushes charge is continuously transferred to the shell and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.